Indian J. Phys. **68B** (4), 341–345 (1994)

I J P B
— an international journal

Finite element analysis of triangular patch microstrip resonator with shape deformation

G M Kalmase
Department of Physics, Science College, Nanded-431 602, India
and
P B Patil*
Department of Physics, Marathwada University, Aurangabad-431 004, India

Received 29 December 1993, accepted 27 April 1994

Abstract : Microstrip resonator with equilateral triangular patch is analysed by Finite Element Method. Further apex of the triangular patch is shifted to the different positions for shape deformation. The resonant frequencies obtained from the Finite Element Method programme, are represented graphically. It is found that when the patch is in the form of an isosceles triangle, the fundamental resonant frequency is highest and for more deformation, a decrease in the fundamental frequency is obtained.

Keywords : Finite element analysis, microstrip resonator, shape deformation

PACS Nos. : 84.40.Yw, 02.70.Dh

1. Introduction

Various structures of microwave resonators such as rectangular, cylindrical, spherical [1,2], have been studied by obtaining exact solutions. Such solutions exist only for limited geometries. Study of ferrite resonators [3] and microstrip resonators [4, 5] have also been studied. Various numerical methods such as variational method [6], finite element method [7] have been used to study the resonators.

Microstrip resonator with equilateral triangular patch is studied earlier by Wolf and Knoppic [8], Helszain and David [9] and Kuester and Chang [10]. Helszain and David [9] have obtained TM mode solutions from duality with TE mode solutions with electric boundaries, whereas Kuester and Chang have obtained the required solutions by the

*For future communication

geometrical theory. In this paper, we have reanalysed these with finite element method and compared the calculated results with the earlier to test the reliability of FEM results. Further, we have considered different triangular patches, which will be the deformed forms of equilateral triangular patch and to which exact solution methods do not apply. We have obtained their resonance frequencies using the FEM Programme. The variation of resonance frequencies are plotted.

2. Statement of the problem

A microwave resonator with conducting triangular patch on a dielectric substrate backed by a grounded conducting plane, inside the region between the patch and ground plane the electric field will satisfy Maxwell's equations, from which we can get the equation

$$\text{Curl Curl } \boldsymbol{E} + K^2\boldsymbol{E} = 0$$

or

$$\text{grad div } \boldsymbol{E} - \nabla^2\boldsymbol{E} + K^2\boldsymbol{E} = 0.$$

Since the medium is charge free div $\boldsymbol{E} = 0$. Hence,

$$\nabla^2\boldsymbol{E} - K^2\boldsymbol{E} = 0.$$

The electric field within the substrate has only Z component and magnetic field has x and y components. Further, if separation between the patch and ground plane *i.e.* thickness of substrate d, is much smaller as compared to wavelength of E.M. waves, then fields will not vary along Z direction. Further, the tangential component of magnetic field at the edge is negligible and we can consider the vertical sides of substrate at the edges of the patch as magnetic walls. Hence, the fields within the resonator corresponding to TM modes will be generated by the equation

$$\nabla^2 E_z - K^2 E_2 = 0, \tag{1}$$

subjected to the boundary condition

$$\left.\frac{\delta E_z}{\delta n}\right|_{B_1} = 0 \tag{2}$$

and

$$E_z\big|_{B_2} = 0, \tag{3}$$

where E_z is z component of $\boldsymbol{E}$, $\frac{\delta}{\delta n}$ represents normal derivatives, B_1 consists of side boundary surfaces and B_2 consists of top patch and part of ground plane below the patch. The domain Ω of resonator is bounded by B_1 and B_2.

3. Variational formulation of the differential equation

To get the expression for the functional Π eq. (1) is multiplied by some weight function V^* and is integrated over the domain of the resonator,

$$\Pi = \iiint_\Omega V^* \text{ div grad } E_z d\Omega - K^2 \iiint_\Omega V^* E_z d\Omega. \tag{4}$$

The first term is of the type S div A and we can replace it by div (SA) – (grad S). A

$$\therefore \Pi = \iiint_{\Omega} -(\text{grad } V^*).(\text{grad } E_z) d\Omega + \iiint_{\Omega} \text{div}\,(V^* \text{ grad } E_z) d\Omega$$

$$- K^2 \iiint_{\Omega} V^* E_z d\Omega. \qquad (5)$$

Application of Gauss Divergence theorem to the second term in the above equation gives

$$\Pi = -\iiint_{\Omega} (\text{grad } V^*)(\text{grad } E_2) d\Omega - K^2 \iiint_{\Omega} V^* E_z d\Omega$$

$$+ \iint V^* \text{ grad } E_z.\mathbf{n}.ds. \qquad (6)$$

The last integral can be written seperately for B_1 and B_2. But it is clear that due to the condition of eq. (2) the integral over B_2 will vanish. The condition (2) is a natural condition while as (3) is an essential condition. Further, it is worth noting that the variation of fields along Z-direction is absent and hence the integration over Z will not contribute to the functional or at the most, it will contribute a constant term which is not important from variational point of view. Hence, we transform the volume integral over the surface which is bounded by the boundary of the patch.

$$\therefore \Pi = \iint \left|(\text{grad } E_z)\right|^2 dxdy - K^2 \iint_s \left|E_z\right|^2 dxdy, \qquad (7)$$

where V is replaced by E_z. For the boundaries B_1, the normal is parallel to Z-axis and grad E_z. $\boldsymbol{n}$ is $\frac{\delta E_z}{\delta Z}$. Since there is no variation of fields along Z-axis, the contribution from the term at B_1 boundaries is zero.

4. Discretisation and finite element formulation

The triangular area of the patch is divided into triangular as well as quadrilateral elements so that patch is divided into 16 sections and the position of quadrilateral and triangular mesh elements are chosen suitably so that in one row of elements only one triangular element appears. We have favoured the quadrilateral elements over the rectangular because triangular elements do not reflect the symmetry eventhough it is there in the entire geometry. The mapping functions assumed for these elements are quadratic in nature and the triangular elements or quadrilateral elements are taken with six or eight nodes respectively.

$$\Pi = \sum_{elm} \left\{E_z^e\right\}^T \left\{S^e\right\} \left\{E_z^e\right\} - K^2 \sum_{elm} \left\{E_z^e\right\}^T \left\{T^e\right\} \left\{E_z^e\right\} \qquad (8)$$

where $$S_{ij}^e = \int \frac{\delta F_i}{\delta x} - \frac{\delta F_j}{\delta x} + \frac{\delta F_i}{\delta y} \frac{\delta F_j}{\delta y} + \frac{\delta F_i}{\delta z} \frac{\delta F_j}{\delta z} \mid dx\,dy \qquad (9)$$

and $$T_{ij}^e = \int F_i F_j \,dx\,dy. \qquad (10)$$

Here F_i is the mapping function due to i^{th} node of an element and integrations are over mesh element surface.

The condition that variation of Π must be zero gives us the matrix equation in terms of assembled matrices S and T as

$$[S]\{E_z\} - K^2[T]\{E_z\} = 0 \tag{11}$$

where $\{E_z\}$ is a set of values of E_z at all nodes of the geometry.

5. Calculation for deformed microwave resonator

Firstly, an isosceles triangular microstrip resonator of sides $1, 1, \sqrt{2}$ cm. is considered. Resonating frequencies are worked out by running the F.E.M. programme. The shape of the triangular patch is then deformed by keeping its base and height the same, but by changing apex position to left at an interval of 0.25 cm (Figure 1). Eigen values are found by FEM programme. The obtained eigenvalues are the values of $\omega^2 \varepsilon \mu$. Considering the substrate with

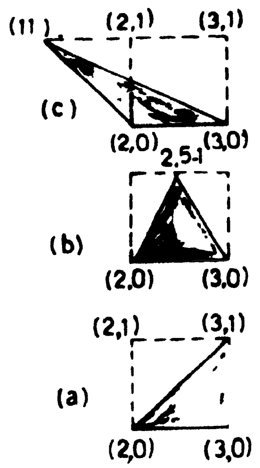

Figure 1. Triangular patch with shape deformation.

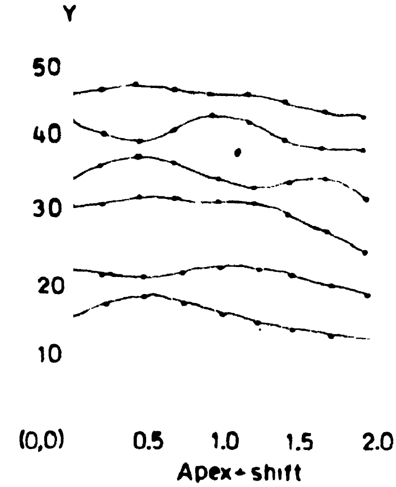

Figure 2. Variation of resonant frequencies with shape deformation of triangular patch.

Table 1. Resonant frequency K^2 in GHz for different positions of vertex of a triangular patch in a planner resonator.

Sr. No.	No deformation	Vertex of triangular patch shifted by							
		0.25 cm	0.5 cm	0.75 cm	1.00 cm	1.25 cm	1.5 cm	1.75 cm	2.00 cm
1	14.99	16.33	17.56	16.53	14.99	13.53	12.19	10.97	9.89
2	21.21	20.55	19.66	20.55	21.21	20.55	19.13	17.61	16.21
3	30.00	30.74	31.57	30.75	30.01	30.54	30.91	25.23	22.87
4	33.54	35.74	37.52	35.79	33.63	31.15	32.51	32.48	30.23
5	42.43	40.35	37.72	40.37	42.66	40.80	37.88	36.88	36.68
6	45.06	45.09	47.10	46.17	45.17	45.00	44.19	41.97	41.38

$\varepsilon = 1$ and $\mu = 1$, the corresponding resonating frequencies are determined. The resonating frequencies for different shapes of the patch are given in Table 1. The variation of the first five frequencies with shift of the apex point of the triangular patch is shown graphically in Figure 2.

6. Conclusions

The Shape deformation of the triangular patch indicates that for some area of triangular patch, the fundamental resonant frequency is highest when the patch is in the form of isosceles triangle. More the change of shape from this, more is the decrease of the fundamental resonant frequency. The higher frequencies however, show increase and decrease of their values with deformation.

References

[1] M L Sisodia and G S Raghuvanshi *Microwave Circuits and Passive Devices* (New Delhi : Wiely-Estern) (1987)

[2] T Iton *IEEE Transaction on Microwave Theory and Techniques* **Vol. MTT-22** p 946 (1974)

[3] Santis petro DE *IEEE Transactions on Microwave Theory and Techniques*, **Vol. MTT-25** p 360 (1977)

[4] S G Pintzos and R Pregla *IEEE Transaction on Microwave Theory and Techniques* **Vol. MTT-26** p 809 (1978)

[5] I Wolff and W Menzel *IEEE Transaction on Microwave Theory and Techniques* p 441 (1975)

[6] J B Davies and F A Fernandez and G Y Phillppoll *IEEE Transaction on Microwave Theory and Techniques* **Vol. 30** p 1975 (1982)

[7] A Konard *IEEE Transaction on Microwave Theory and Techniques* **Vol. MTT-34** p 224 (1986)

[8] I Wolff and N Knoppik *IEEE Transaction on Microwave Theory and Techniques* **Vol. MTT-22** p 857 (1974)

[9] J Helszajn and S J David *IEEE Transaction on Microwave Theory and Techniques* **Vol. MTT-26** p 95 (1978)

[10] E F Kuester and D C Chang *IEEE Transaction on Microwave Theory and Techniques* **Vol. AP-31** p 27 (1983)